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A new model to explain the forces between moving charges

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Abstract: A model is presented which explains the forces between charges in uniform motion in a novel way, without reference to magnetism or special relativity. Building on Coulomb's law of electrostatics, the model explains the magnitude and the direction of such forces in terms of changes in the flow of information from one charge to the other resulting from their acceleration histories. The model enables a deeper understanding of the forces between moving charges than classical electromagnetism, which notably leaves the origin of the magnetic field unexplained. It also offers fresh insights into the foundations of special relativity. In particular, the model provides a simple and direct explanation of how the forces between charges in uniform motion are transformed in a way that is consistent with length contraction. The model also implies that, in the framework of special relativity, events with equal time coordinates are in general not simultaneous. Some implications of this finding for causality, the "block universe" idea and the Andromeda paradox are briefly discussed. © *2018 Physics Essays Publication*.
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Résumé: Un modèle est présenté qui propose une nouvelle explication des forces entre des charges en mouvement uniforme, sans avoir recours au magnétisme ou à la relativité restreinte. À partir de la loi de Coulomb de l'électrostatique, le modèle explique la magnitude et la direction de telles forces par des modifications du flux d'informations d'une charge à l'autre résultant de l'évolution passée de leurs accélérations respectives. Le modèle améliore la compréhension des forces entre les charges en mouvement par rapport à l'électromagnétisme classique, qui notamment n'explique pas l'origine du champ magnétique. De plus, il offre de nouvelles perspectives sur les fondements de la relativité restreinte. En particulier, le modèle fournit une explication simple et directe de la manière dont les forces entre des charges en mouvement uniforme se transforment conformément à la contraction des longueurs. De plus, le modèle implique qu'en général, dans le cadre de la relativité restreinte, les événements possédant les mêmes coordonnées dans le temps ne sont en fait pas simultanés. Quelques implications de ce résultat pour la causalité, l'idée de "l'univers-bloc" et le paradoxe d'Andromède sont brièvement discutées.

Key words: Electromagnetism; Special Relativity; Length Contraction; Clock Synchronization; Andromeda Paradox.

I. INTRODUCTION

In classical electromagnetism, the equations describing the interactions between moving charges involve nothing but parameters describing the charges' magnitudes, their relative positions and their states of motion, the speed c at which electric disturbances travel in empty space, and the constant ε_0 from Coulomb's law of electrostatics.[1] The question thus arises whether those equations can be developed from a model of the interactions between charged particles based on nothing but Coulomb's law and the fact that electric disturbances travel at c. In this article, I will present such a model for the case of two point charges moving at constant velocities.

Let Σ be an inertial frame of reference in which, in the absence of any medium, electric disturbances travel in isotropic conditions from bodies at rest and clocks have been Einstein-adjusted.[2] Experience shows that in those conditions electric disturbances travel at the speed c in all directions in Σ. Let q_1 and q_2 be two point charges at rest in Σ. According to Coulomb's law,

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}_{12}, \tag{1}$$

the magnitude of the force between the two charges depends only on their magnitudes q_1 and q_2 and on the distance r between them, and the force acts in the direction of the unit vector $\hat{\mathbf{r}}_{12}$ pointing from q_1 to q_2. Now consider the force on q_2 if q_1 moves at the constant velocity $\mathbf{u}$ and q_2 moves at the constant velocity $\mathbf{v}$ as measured by an observer at rest in Σ. Let the force be measured in the rest frame of q_2, for example, by a spring balance co-moving with q_2. In classical electromagnetism, it is straightforward to calculate this force using the concepts of the electric field and the magnetic field and the transformations of special relativity (see, for example, Ref. 1). To do so, for the purposes of this article, it is convenient to choose a Cartesian coordinate system such that q_2 is at the origin, the velocity $\mathbf{v}$ points in the direction of the x-axis, and the vector $\mathbf{r}_{12}$ from q_1 to q_2 lies in the xy-plane (Fig. 1).

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In terms of this coordinate system, the magnitude of the force on q_2 can be written as

$$F = \frac{q_1 q_2 \left(1 - \frac{u^2}{c^2}\right) \sqrt{\left(1 + \frac{vu}{c^2} \sin\theta \sin\gamma \cos\delta\right)^2 - \frac{v^2}{c^2} \cos^2\theta \left(1 - \frac{u^2}{c^2} \sin^2\alpha\right)}}{4\pi\varepsilon_0 r^2 \sqrt{1 - \frac{v^2}{c^2}} \left(1 - \frac{u^2}{c^2} \sin^2\alpha\right)^{\frac{3}{2}}} \tag{2}$$

and its direction in Σ, i.e., the direction of the co-moving spring balance as measured in Σ, is given by the vector

$$\begin{pmatrix} \cos\theta\left(1 - \frac{v^2}{c^2}\right) \\ \sin\theta + \frac{uv}{c^2}\sin\gamma\cos\delta \\ \frac{uv}{c^2}\cos\theta\sin\delta \end{pmatrix} \tag{3}$$

with $\theta := \measuredangle(\mathbf{v};\, \mathbf{r}_{12})$ $\gamma := \measuredangle(\mathbf{r}_{12};\, \mathbf{u}_{xy})$ $\delta := \measuredangle(\mathbf{u}_{xy};\, \mathbf{u})$ and $\alpha := \measuredangle(\mathbf{r}_{12};\, \mathbf{u})$.

The angle α is not an independent parameter and has been introduced for convenience only. It is related to γ and δ as follows:

$$\cos\alpha = \cos\gamma\cos\delta. \tag{4}$$

In this article, I will develop a model of the exchange of information between charged particles that explains how Coulomb's law (1) is transformed into Eq. (2) and (3) in the case of two charges moving at constant velocities in Σ. I will do so without any reference to magnetism or special relativity.

The significance of the model presented here lies in the fact that it provides a simple and coherent explanation of the forces between moving charges while classical electromagnetism provides no such explanation. It is, for example, not clear in classical electromagnetism why there should be magnetic forces on moving charges that act in a direction that is perpendicular to the velocity of those charges. Indeed, as I will show later, there are good reasons for saying that in general there are no such forces between moving charges. It is also unclear why the magnetic constant $\mu_0 = 1/\varepsilon_0 c^2$ in magnetic field equations includes the speed c at which electric disturbances propagate. In classical electromagnetism, this is an unexplained empirical finding. Finally, the fact that magnetic forces can turn into purely electric forces if considered from a different frame of reference underlines that, in classical electromagnetism, the magnetic field does not correspond to any physical reality but is merely part of a useful mathematical formalism. This has prompted some authors to declare that the magnetic field "is a fiction" and that a theory of "electromagnetism without magnetism" should be developed.[3] For overviews of such efforts, see Refs. 3 and 4.

In the model presented in this article, there is no magnetic field and there are no magnetic forces. Instead, the forces between moving charges are the result of distortions in the electric properties of the space surrounding charged particles brought about by the particles' past accelerations. The directions in which information about these distortions is communicated to other moving charges determine the direction of the forces on those charges. The speed c makes an appearance in the model solely as a result of the fact that electric information is transmitted at c.

Some authors have suggested that magnetic forces in the interactions between charged particles are best understood as "relativistic effects" (see, for example, Refs. 5–7). However, special relativity does not make the concepts of the magnetic field and magnetic forces redundant: In the framework of special relativity, magnetic fields are still said to be present in any inertial frame of reference in which charged particles move. Since in special relativity any inertial frame of reference is as good as any other for the description of electromagnetic phenomena, the origin of magnetic fields and magnetic forces remains unexplained in classical electromagnetism even if special relativity is taken into account.

Another reason why special relativity does not provide a satisfactory explanation of magnetic effects between moving charged particles is that it is itself in need of an explanation. For example, how does length contraction by the relativistic factor come about? We know that a suitable transformation of the forces between charged particles as a result of past acceleration is a necessary condition for length contraction to occur. The model presented in this article provides a very simple and direct explanation of how such a transformation comes about. The model thus goes some way toward explaining the physical basis of special relativity—a quest which some authors believe has been neglected in the

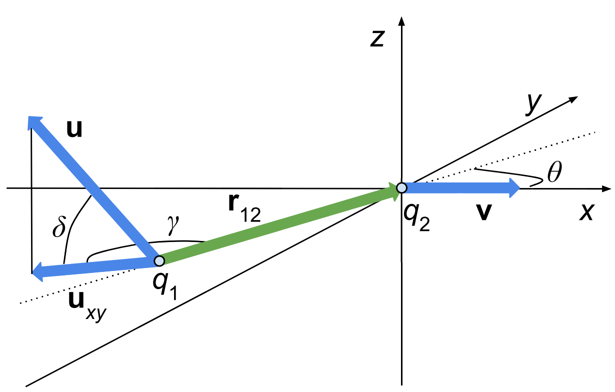

FIG. 1. (Color online) For any two point charges q_1 and q_2 moving at constant velocities $\mathbf{u}$ and $\mathbf{v}$ in Σ, a Cartesian coordinate system can be chosen such that q_2 is at the origin, the velocity $\mathbf{v}$ points in the direction of the x-axis, and the vector $\mathbf{r}_{12}$ from q_1 to q_2 lies in the xy-plane. The situation is fully described by specifying q_1 and q_2, r, u and v and the following angles: $\theta := \measuredangle(\mathbf{v}; \mathbf{r}_{12})$, $\gamma := \measuredangle(\mathbf{r}_{12}; \mathbf{u}_{xy})$, and $\delta := \measuredangle(\mathbf{u}_{xy}; \mathbf{u})$.

traditional treatment of that theory (see Ref. 8 for an overview and Ref. 9 for a similar argument).

The model also sheds new light on the much-debated concept of simultaneity in special relativity (see Ref. 10 for an overview of those debates). It implies that the conditions in which electric disturbances propagate are not isotropic in all inertial frames of reference and that therefore in general Einstein's clock adjustment procedure as described by him in 1905 (Ref. 2) is not a synchronization procedure. This means that events with equal time coordinates as determined by Einstein-adjusted clocks are in general not simultaneous.

The significance of the model presented in this article is thus threefold:

First, it explains the forces between charges in uniform motion in a simple and coherent manner while classical electromagnetism provides no such explanation.

Second, it can be used to show very simply and directly that the forces between charges in uniform motion are consistent with the length contraction of moving bodies in inertial frames of reference in which clocks have been Einstein-adjusted.

Third, it implies that in general events with equal time coordinates as determined by Einstein-adjusted clocks are not simultaneous.

I will now first present the new model, which I will call the "sphere model." I will then discuss an example which illustrates how the sphere model explains the forces between charges moving at constant velocities more simply and more clearly than classical electromagnetism. I will conclude by discussing how the model relates to special relativity.

II. OUTLINE OF THE MODEL

I will make the following assumptions:

(1) Locally at any point in time and space there is an inertial frame of reference Σ in which electric disturbances, such as light waves, propagate from bodies at rest in isotropic conditions. This ensures that Einstein-adjusted clocks at rest in Σ are synchronized.[2,11] Note that this first assumption is much weaker than the assumption made in classical theory that locally such isotropy pertains in any inertial frame of reference.

(2) The field of a point charge at rest in Σ can be represented by a series of concentric sphere surfaces, as shown in cross section in Fig. 2. By the "field of a point charge" I mean the properties of the space surrounding the charge which are responsible for the electric forces on other charges caused by the presence of the point charge. The assumption that this field can be represented by a series of concentric sphere surfaces is consistent with Coulomb's law and also with the assumed isotropy of the conditions in which electric disturbances propagate from bodies at rest in Σ.

(3) For point charges at rest in Σ, the distance Δr between any two neighboring sphere surfaces is the same. The precise size of that distance is irrelevant in the sphere model, but I will assume that it is extremely small compared to the distances between charged particles considered in classical electromagnetism. I will also assume that the size of the point charge is extremely small compared to Δr.

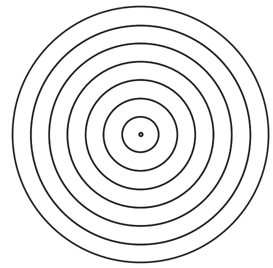

FIG. 2. Schematic representation of the field of a point charge at rest in Σ in the sphere model (cross section). The point charge is represented by the small innermost sphere.

(4) Information about a point charge in Σ continually spreads outward from the charge in all directions. This assumption is consistent with the concept of retardation in classical electromagnetism. In classical theory, the magnitude, position, and velocity of a charge affect other charges with a certain delay, given by the time it takes for information on these parameters traveling at the speed of light to reach the other charges. The same parameters are also relevant in the sphere model, as will be explained in more detail in Secs. III–V.

(5) Information about a point charge in Σ always takes the same time Δt to traverse the space between two neighboring sphere surfaces. This can be regarded as the fundamental sphere model law. It means that information spreading out from a stationary point charge always travels on information sphere surfaces expanding at one and the same speed $c := \Delta r / \Delta t$ in Σ.

(6) The sphere surfaces associated with a point charge always move at the velocity communicated to them by information sphere surfaces associated with that charge that pass over them. As a result, information spreading out from moving point charges also travels on information sphere surfaces expanding at c in Σ, and changes in the state of motion of a point charge always ripple through the sphere surfaces associated with that charge at the speed c in Σ.

Together these six assumptions ensure that the sphere model is consistent with the observed behavior of electric disturbances spreading out from point charges at rest in Σ as a result of a sudden acceleration. In the sphere model, what happens in the event of such an acceleration is shown in cross section in Fig. 3 for three successive times t_0, t_1, and t_2.

In Fig. 3, the highlighted surfaces indicate the location of an electric disturbance, marked by a jump in sphere surface densities, spreading outward through the sphere surfaces at the speed c as a result of a sudden acceleration of a point charge to a velocity $\mathbf{v}$ at the time $t = t_0$. Since electric information always travels at c in Σ even if it originates from a charge that already moves at a velocity $\mathbf{v}$ in Σ, sphere surface disturbances also invariably travel at c in Σ, in accordance

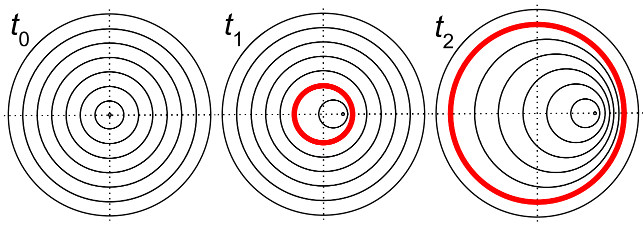

FIG. 3. (Color online) Changes in a point charge's sphere surface arrangement occur when the charge is briefly accelerated to a velocity $\mathbf{v}$ in Σ. The changes cause an electric disturbance (located at the highlighted sphere surfaces) that spreads outward at the speed c in Σ.

with observations. This is illustrated in Fig. 4 for a charge accelerated from a velocity $\mathbf{v}$ at $t = t_0$.

The highlighted sphere surfaces again show the location of an electric disturbance spreading outward through the sphere surfaces at the speed c in Σ. In the sphere model, it is assumed that any two charges moving at one and the same constant velocity in Σ are surrounded by the same kind of sphere arrangement, as a result of the charges' respective acceleration histories.

I will now consider the interaction of a first charge q_1 traveling at $\mathbf{u}$ in Σ with a second charge q_2 traveling at $\mathbf{v}$ in Σ, as shown in Fig. 5. The velocity $\mathbf{v}$ of q_2, which might be in a direction into or out of the plane defined by the page, is not shown in Fig. 5. This is because, as will become clear in the subsequent discussion, for some aspects of the interaction of q_1 with q_2 the velocity of q_2 is irrelevant.

In Fig. 5, A is at the centre of the q_1 sphere surface on which q_2 is located. This means that the information about q_1 arriving at q_2 at the time t_1 shown in Fig. 5 originated from A at the "retarded time" t_0, when q_1 was at A. This information travelled from A to B at the speed c in the same period of time in which q_1 travelled from A to C at the speed u.

From the point of view of q_1, information leaving A at t_0 and arriving in B at t_1 travels in the direction defined by the vector $c\mathbf{n}_1^< - \mathbf{u} = c\left(\mathbf{n}_1^< - \mathbf{u}/c\right)$, which is parallel to $\overline{DB}$. Similarly, from the point of view of q_1, information leaving A at t_0 and arriving in D at t_1 travels in the direction defined by the vector $c\mathbf{n}_1^> - \mathbf{u} = c\left(\mathbf{n}_1^> - \mathbf{u}/c\right)$, which is also parallel to $\overline{DB}$. It is not just that the information *seems* to travel in those two directions from the point of view of q_1, it really does so in a physical sense: If q_1 were attached to a rod moving at $\mathbf{u}$ and oriented in the direction $\overline{DB}$ in Σ, then q_1 information traveling from A to B and from A to D would travel along that rod. This means that information reaching q_2 traverses the q_1 sphere surface arrangement in the direction $\overline{DB}$.

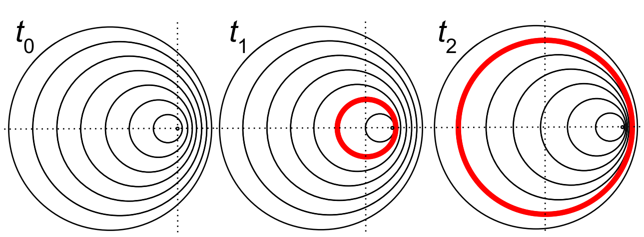

FIG. 4. (Color online) A brief acceleration of a point charge already in motion in Σ causes an electric disturbance (located at the highlighted sphere surfaces) to spread through the sphere surfaces at the speed c in Σ.

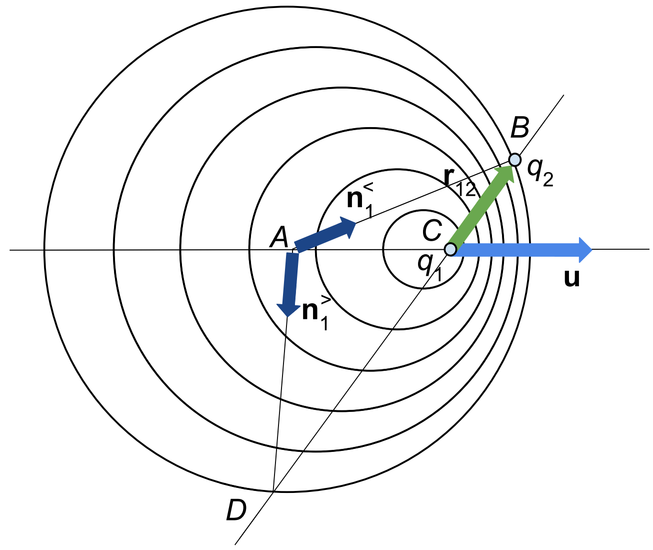

FIG. 5. (Color online) A is at the centre of the q_1 sphere surface on which q_2 is located. The sphere surface density of q_1 spheres surfaces along $\overline{CB}$ is different from the sphere surface density along $\overline{CD}$. The directions from A to B and from A to D are represented by the unit vectors $\mathbf{n}_1^<$ and $\mathbf{n}_1^>$.

In addition, the q_1 information leaving A for B at the retarded time t_0 can be divided into two classes:

(a) Information about q_1 on the side of the line $\overline{AC}$ on which q_2 is located at the time t_1. I will call this the "near (<) side" of q_1.
(b) Information about q_1 on the other side of the line $\overline{AC}$. I will call this the "far (>) side" of q_1.

It is an essential aspect of the sphere model that information about the q_1 sphere arrangement from both the near side and the far side of q_1 at the retarded time t_0 must be taken into account to determine the force on q_2 at the time t_1. In the sphere model, such information is encoded in two sphere model parameters which depend on the velocity of q_1: The "density factor" and the "angle factor." The way in which q_1 information is communicated to q_2 is encoded in a third sphere model parameter, the "frequency factor," which depends on the velocity of q_2 as well as that of q_1.

III. THE DENSITY FACTOR

In Fig. 5, the sphere surface densities along the line $\overline{DB}$ on the near and the far side of q_1 are constant but different from each other. Let $\lambda_1^<$ be the near-side density and $\lambda_1^>$ the far-side density. The factors by which these densities are different from the sphere surface density $\lambda_\Sigma := 1/\Delta r$ when q_1 is at rest in Σ are $d_1^< := \lambda_1^</\lambda_\Sigma$ and $d_1^> := \lambda_1^>/\lambda_\Sigma$. It turns out that, in the sphere model, the parameter that matters is the geometric mean of these two quantities

$$\overline{d_1} := \sqrt{d_1^< d_1^>}. \tag{5}$$

I will call $\overline{d_1}$ the density factor. The density factor is thus the mean factor by which the density of q_1 sphere surfaces in the direction of the line connecting q_1 and q_2 at the time t_1 is different compared to when q_1 is at rest in Σ. The density

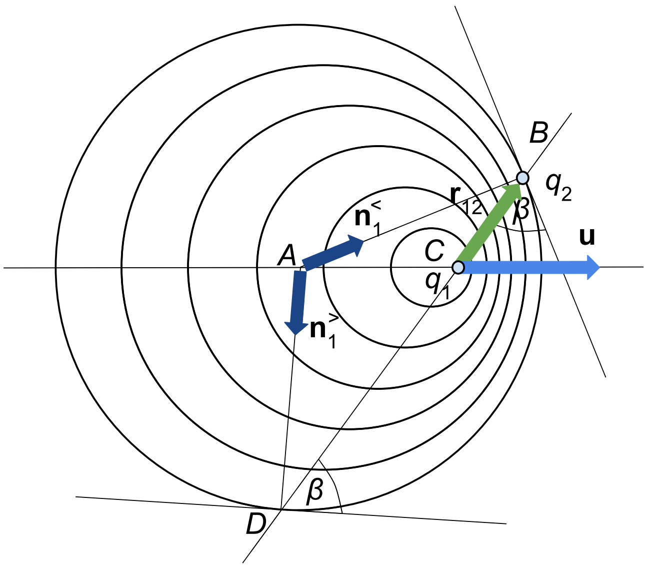

FIG. 6. (Color online) The angle β at which the line connecting $q_1$and q_2 intersects q_1 sphere surfaces in the plane defined by $\mathbf{u}$ and $\mathbf{r}_{12}$ is the same for the near side and the far side of q_1. The directions from A to B and from A to D are again represented by the unit vectors $\mathbf{n}_1^<$ and $\mathbf{n}_1^>$.

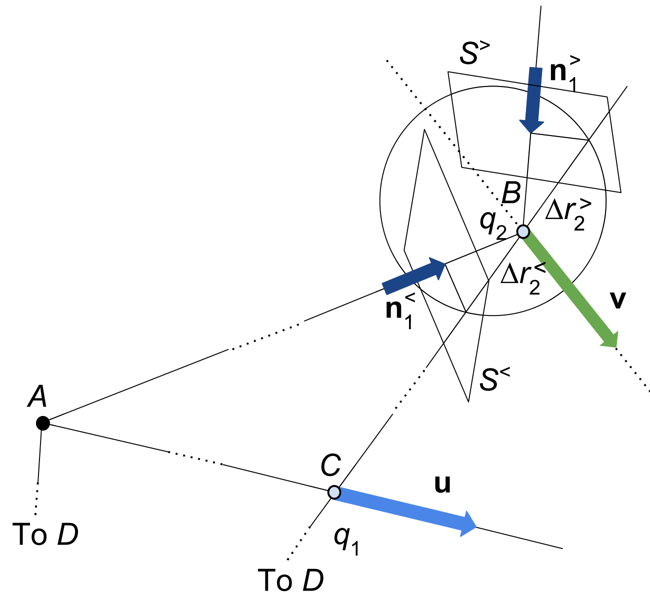

FIG. 7. (Color online) This is a representation of the situation in the immediate vicinity of q_2. The situation is three-dimensional since in general $\mathbf{u}$, $\mathbf{v}$ and the line connecting q_1 and q_2 do not lie in the same plane. In the sphere model, q_1 information reaches q_2 locally in B from two directions, given by the unit vectors $\mathbf{n}_1^<$ and $\mathbf{n}_1^>$. The information is carried by information sphere surfaces $S^<$ and $S^>$, which take different times to reach q_2 from their locations shown in the figure. To calculate those times, $S^<$ and $S^>$ can locally be approximated by planes moving at the speed c toward the location of q_2 at the time t_1.

factor is also a local property of the q_1 sphere surface arrangement in A at the retarded time t_0. In the sphere model, it is this property which is communicated to q_2 at the time t_1 in B, where it influences the force on q_2.

IV. THE ANGLE FACTOR

Another sphere model parameter that is different from the static case when q_1 moves at $\mathbf{u}$ is the angle β at which the line connecting q_1 and q_2 intersects q_1 sphere surfaces in the plane defined by $\mathbf{u}$ and $\mathbf{r}_{12}$. This is shown in Fig. 6, where A is again at the centre of the q_1 sphere surface on which q_2 is located. The angle β is the same for the near side and the far side of q_1 since the triangle ABD is isosceles. It is also the same for any q_1 sphere surface since the basic geometry is the same in each case.

In Fig. 6, it can be seen that, since $\beta < \pi/2$, the trajectory taken by information traveling from q_1 to B from one q_1 sphere surface to the next is longer than the local perpendicular distance between those sphere surfaces (defined as the distance between the tangent planes to those surfaces along the line $\overline{DB}$). The factor by which it is longer for any two sphere surfaces is the same because the basic geometry is the same. If the factor by which it is longer on the near side of q_1 is $e_1^< = 1/\cos\measuredangle\left(\mathbf{n}_1^<;\mathbf{n}_1^< - \mathbf{u}/c\right) = 1/\sin\beta$ and the factor on the far side is $e_1^> = 1/\cos\measuredangle\left(\mathbf{n}_1^>;\mathbf{n}_1^> - \mathbf{u}/c\right) = 1/\sin\beta$, we can again define a mean factor $\overline{e_1}$ as

$$\overline{e_1} := \sqrt{e_1^< e_1^>}. \tag{6}$$

I will call $\overline{e_1}$ the angle factor. The angle factor is thus the mean factor by which the path between neighboring q_1 sphere surfaces along the line connecting q_1 and q_2 is different from the local perpendicular distance between the sphere surfaces. Once again, the angle factor $\overline{e_1}$ is also a local property of the q_1 sphere surface arrangement in A at the retarded time t_0. Information about $\overline{e_1}$ thus reaches q_2 at the time t_1 in B.

V. THE FREQUENCY FACTOR

In Fig. 7, the focus is shifted to the immediate vicinity of point B in Fig. 6 at the time t_1. This is because the third parameter to be introduced here concerns the way in which the q_1 information arriving at q_2 interacts with q_2. From the sphere model, we know that the q_1 information arriving at q_2 is located on q_1 information sphere surfaces which expand at c in Σ. Any one such q_1 information sphere surface will cross successive q_2 sphere surfaces along the line connecting q_1 and q_2 in the immediate vicinity of B at a particular "information transmission rate," which is liable to be different from the rate $\zeta_\Sigma := 1/\Delta t$ in the static case. It is plausible for the factor by which these rates are different to influence the force on q_2, and it turns out that this is indeed the case. Before we determine this factor we need to remember that we have to consider both near- and far-side parameters.

The near- and far-side unit vectors $\mathbf{n}_1^<$ and $\mathbf{n}_1^>$ shown in Fig. 7 are the same as those shown in Figs. 5 and 6. In the sphere model, it is assumed that information about both $\mathbf{n}_1^<$ and $\mathbf{n}_1^>$ is communicated from A to q_2 such that q_1 information appears to arrive at q_2 from those two directions, rather than just from the direction given by $\mathbf{n}_1^<$. Each of these two vectors is locally associated with a q_1 information sphere surface. In Fig. 7 these are indicated schematically as $S^<$ and $S^>$. The line connecting q_1 and q_2 intersects the q_1 information sphere surfaces $S^<$ and $S^>$ at the distance $\Delta r_2^< = 1/\lambda_2^<$ and $\Delta r_2^> = 1/\lambda_2^>$ from q_2, respectively. Information carried by $S^<$ and $S^>$ takes a particular time, $t^<$ and $t^>$, to reach q_2, which we can use to define the respective q_1 information transmission rates $\zeta_1^< := 1/t^<$ and $\zeta_1^> := 1/t^>$. We can now define $f_1^<$ and $f_1^>$ as the factors by which the respective rates $\zeta_1^<$ and $\zeta_1^>$ are different from the rate $\zeta_\Sigma = 1/\Delta t$ that applies when both q_1 and q_2 are at rest in Σ.

What matters for the magnitude of the force on q_2 is, once again, the geometric mean

$$\overline{f_1} := \sqrt{f_1^< f_1^>}. \tag{7}$$

I will call $\overline{f_1}$ the frequency factor. The frequency factor is thus the mean factor by which the q_1 information transmission rate is different from the rate that applies in the static case. The frequency factor is the most complex of the three parameters introduced here because it depends on both $\mathbf{u}$ and $\mathbf{v}$.

VI. MAIN FINDINGS

Having defined three relevant sphere model parameters and the direction vectors $\mathbf{n}_1^<$ and $\mathbf{n}_1^>$, I am now in a position to formulate my main findings. My first main finding is that, in the situation described in Section I, the magnitude of the force on q_2 as measured by a co-moving spring balance can be expressed as follows:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\frac{\overline{e_1}^2}{\overline{d_1}^2}\overline{f_1}. \tag{8}$$

My second main finding is that the direction of the force on q_2 as measured by a co-moving spring balance is parallel to the vector

$$e_2^<\left(\widehat{\mathbf{n}_1^< - \frac{\mathbf{v}}{c}}\right) - e_2^>\left(\widehat{\mathbf{n}_1^> - \frac{\mathbf{v}}{c}}\right), \tag{9}$$

where, in analogy to the q_1 near- and far-side angle factors $e_1^<$ and $e_1^>$, the coefficients $e_2^<$ and $e_2^>$ are the q_2 near- and far-side angle factors $e_2^< := 1/\cos\measuredangle(\mathbf{n}_1^<; \mathbf{n}_1^< - \mathbf{v}/c)$ and $e_2^> := 1/\cos\measuredangle(\mathbf{n}_1^>; \mathbf{n}_1^> - \mathbf{v}/c)$, respectively. It is plausible for the direction of the force to be a combination of the unit vectors $\widehat{\mathbf{n}_1^< - \mathbf{v}/c}$ and $\widehat{\mathbf{n}_1^> - \mathbf{v}/c}$ since these vectors represent the directions from which q_1 information arrives at q_2 from the point of view of q_2. This is true in the same physical sense as explained previously for q_1 information leaving q_1 for q_2 from the point of view of q_1.

In the sphere model, the changes to Coulomb's law in the case of two moving charges are thus the result of changes in the arrangement of sphere surfaces surrounding the charges and associated changes in the flow of information between the charges in Σ. They are not the result of magnetism as classically conceived or of the transformations of special relativity.

It is straightforward to express $\overline{d_1}$, $\overline{e_1}$, $\overline{f_1}$, $\mathbf{n}_1^<$, and $\mathbf{n}_1^>$ in terms of the locations and velocities of the charges q_1 and q_2 defined in Sec. I. The results of these calculations are as follows:

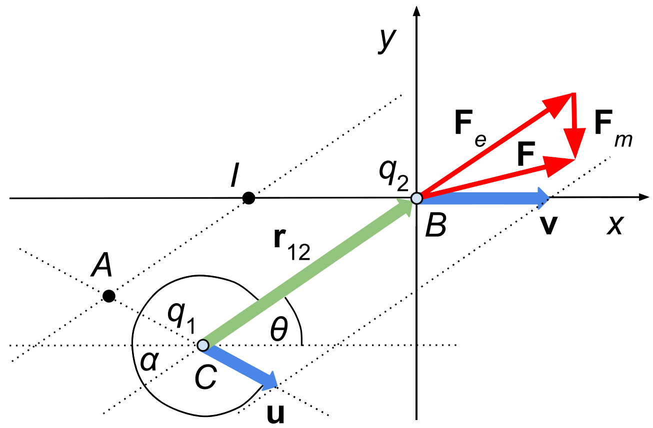

FIG. 8. (Color online) In classical electromagnetism, the electric and magnetic forces acting on q_2 when $(\mathbf{u}-\mathbf{v}) \parallel \mathbf{r}_{12}$ combine in such a way that the overall force is not parallel to $\mathbf{r}_{12}$. A and I are the retarded positions of q_1 and q_2, respectively. Information that leaves q_1 in A at the retarded time t_0 thus arrives at q_2 at the time t_1 shown in the illustration.

$$\overline{d_1} = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}, \tag{10}$$

$$\overline{e_1} = \frac{1}{\sin\beta} = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}\sin^2\alpha}}, \tag{11}$$

$$\overline{f_1} = \frac{\sqrt{\left(1 + \frac{uv}{c^2}\sin\theta\sin\gamma\cos\delta\right)^2 - \frac{v^2}{c^2}\cos^2\theta\sin^2\beta}}{\sin\beta\sqrt{1 - \frac{v^2}{c^2}}}, \tag{12}$$

$$\mathbf{n}_1^< = \begin{pmatrix} -\frac{u}{c}\sin\theta\sin\gamma\cos\delta + \cos\theta\sin\beta \\ \frac{u}{c}\cos\theta\sin\gamma\cos\delta + \sin\theta\sin\beta \\ \frac{u}{c}\sin\delta \end{pmatrix}, \tag{13}$$

$$\mathbf{n}_1^> = \begin{pmatrix} -\frac{u}{c}\sin\theta\sin\gamma\cos\delta - \cos\theta\sin\beta \\ \frac{u}{c}\cos\theta\sin\gamma\cos\delta - \sin\theta\sin\beta \\ \frac{u}{c}\sin\delta \end{pmatrix}. \tag{14}$$

From these results, it can be seen by simple comparison that Eq. (8) is equivalent to the classical expression (2). Using Eqs. (13) and (14), it can also be shown that the vector given in (9) is parallel to the classical result (3).

VII. CONCEPTUAL CLARITY

An illustration of the conceptual advantages of the sphere model is provided by the case of $(\mathbf{u}-\mathbf{v}) \parallel \mathbf{r}_{12}$. The principle of relativity tells us that in this situation, shown in Fig. 8, any effect on q_2 must be in the direction of the line connecting the two charges, since their relative movement is along that line. We would thus expect a spring balance co-moving with q_2 to show a force in the direction of $\mathbf{r}_{12}$ in Σ.

In classical electromagnetism, the force on q_2 is given by the Lorentz force law

$$\begin{aligned}\mathbf{F} &= q_2(\mathbf{E} + \mathbf{v}\times\mathbf{B})\\ &= \mathbf{F}_e + \mathbf{F}_m\\ &= \frac{q_1 q_2\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}\begin{pmatrix}\cos\theta\\ \sin\theta\left(1-\frac{v^2}{c^2}\right)\\ 0\end{pmatrix}.\end{aligned} \quad (15)$$

As can be seen in Fig. 8 as well as in Eq. (15), it turns out that this force is *not* in the direction of $\mathbf{r}_{12}$. This means that the decomposition of the electromagnetic force into an electric component determined by the electric field and a magnetic component determined by the magnetic field does not correspond to anything in the observed phenomena. In order to obtain the direction of the force as measured by a co-moving spring balance, which is also the direction in which q_2 would be accelerated in Σ if it were released from the spring balance, a relativistic weighting factor has to be introduced: The component of the force that is parallel to the velocity $\mathbf{v}$ of q_2, which in this case is the x-component, must be multiplied by $1 - v^2/c^2$. Taking into account the relativistic weighting factor, the direction of the force as measured by a co-moving spring balance finally turns out to be parallel to $\mathbf{r}_{12}$.

To obtain the magnitude of the force as shown by the spring balance, we have to perform a relativistic force transformation to calculate the force $\mathbf{F}'$ in the rest frame of q_2. Exploiting the fact that in the situation under consideration $u\sin\alpha = -v\sin\theta$, we obtain

$$\begin{aligned}|\mathbf{F}'| &= \frac{q_1 q_2\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}\left|\begin{pmatrix}\cos\theta\\ \sin\theta\sqrt{1-\frac{v^2}{c^2}}\\ 0\end{pmatrix}\right|\\ &= \frac{q_1 q_2\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{u^2}{c^2}\sin^2\alpha\right)}.\end{aligned} \quad (16)$$

In terms of classical electromagnetism alone, it is unclear why the force needs to be transformed in this way.

Now let me consider how the magnitude and direction of the force on q_2 are explained in the sphere model. The correct direction of the force can be read off Fig. 9 immediately since it is clear that both $\mathbf{n}_1^< - \mathbf{v}/c$ and $\mathbf{n}_1^> - \mathbf{v}/c$ are parallel to $\mathbf{r}_{12}$. This can of course also be calculated explicitly using Eqs. (13) and (14), again exploiting the relationship $u\sin\alpha = -v\sin\theta$

$$\mathbf{n}_1^< - \frac{\mathbf{v}}{c} = \left(\sin\beta - \frac{v}{c}\cos\theta\right)\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}, \quad (17)$$

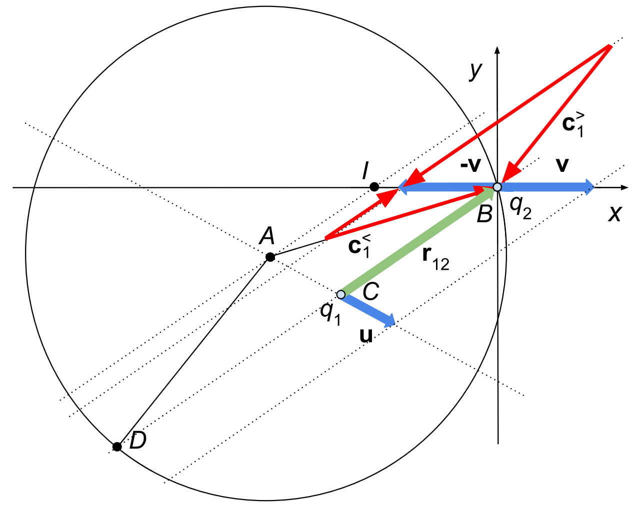

FIG. 9. (Color online) The situation shown here is the same as in Fig. 8 but with the addition of relevant sphere model elements. A, B, C, D, and I are defined as before, and $\mathbf{c}_1^< = c\mathbf{n}_1^<$ and $\mathbf{c}_1^> = c\mathbf{n}_1^>$ indicate the directions from which information originating from A arrives at q_2 from the point of view of an observer at rest in Σ. From the point of view of q_2, the information arrives from the directions $\mathbf{c}_1^< - \mathbf{v} = c(\mathbf{n}_1^< - \mathbf{v}/c)$ and $\mathbf{c}_1^> - \mathbf{v} = c(\mathbf{n}_1^> - \mathbf{v}/c)$, which are both parallel to $\mathbf{r}_{12}$.

$$\mathbf{n}_1^> - \frac{\mathbf{v}}{c} = -\left(\sin\beta + \frac{v}{c}\cos\theta\right)\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}. \quad (18)$$

Hence, as per (9), the overall direction of the force is also parallel to $\mathbf{r}_{12}$. In the sphere model this is the correct direction because, from the point of view of q_2, information about q_1 is arriving at q_2 from that direction.

Finally, the magnitude of the force is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\frac{\overline{e_1}^2}{\overline{d_1}^2}\overline{f_1} = \frac{q_1 q_2\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{u^2}{c^2}\sin^2\alpha\right)}. \quad (19)$$

The magnitude and direction of the force are thus the result of modifications in the q_1 and q_2 sphere arrangements and associated changes in the flow of information from q_1 to q_2, as captured in the sphere model parameters $\overline{d_1}$, $\overline{e_1}$, and $\overline{f_1}$ and the direction vectors $\mathbf{n}_1^<$ and $\mathbf{n}_1^>$. The speed c enters into the direction and the magnitude of the force solely because electric information in Σ is transmitted at the speed c. This is highly plausible and contrasts with the enigmatic role played by c in magnetic field equations in classical electromagnetism.

VIII. RELATIONSHIP WITH SPECIAL RELATIVITY

There are two ways in which the sphere model is linked to special relativity. The first is that it provides a simple and direct explanation of the forces between moving charges that is consistent with length contraction.

In the sphere model, if two stationary charges q_1 and q_2 are separated by r in Σ and are then both accelerated in the direction of $\mathbf{r}_{12}$ to the same velocity $\mathbf{v}$, then as per Eq. (19)

the magnitude of the force between them as measured by co-moving spring balances is

$$F = \frac{q_1 q_2 \left(1 - \frac{v^2}{c^2}\right)}{4\pi\varepsilon_0 r^2}. \tag{20}$$

Consequently, if the force between the moving q_1 and q_2 as measured by co-moving spring balances is to be the same as it was between the stationary q_1 and q_2, then the distance between them in terms of Σ coordinates must be reduced by the relativistic factor $\sqrt{1 - v^2/c^2}$.

The sphere model thus provides a partial explanation of length contraction. The sphere model also implies the independence of the speed of light from the speed of the source of the light in Σ. Together with length contraction, this implies time dilation by the relativistic factor in Σ, provided movement in Σ affects the rate at which any kind of clock ticks in the same way. As shown, for example, in Ref. 12, from length contraction and time dilation in Σ the whole apparatus of special relativity follows, including the constancy of the speed of light for any observer using Einstein-adjusted clocks.

The second way in which the sphere model is linked to special relativity is that it sheds new light on the concept of simultaneity in that theory. It is widely acknowledged that the definition of time coordinates in special relativity is, at least in part, a matter of convention (see, for example, the discussions of this issue in Refs. 2 and 12–15). Indeed, any community of people, such as physicists, is free to adjust clocks any way they like. As for example explained in Refs. 13, 14, and 16, physicists have good reasons to adopt Einstein's clock adjustment procedure using light signals: It leads to the familiar symmetric laws of physics in all inertial frames of reference, which are conveniently connected via the Lorentz transformations.

What is perhaps less widely appreciated is that not every clock adjustment procedure is a synchronization procedure. One of the conditions that must be met for a clock adjustment procedure using signals to qualify as a synchronization procedure is the isotropy of the conditions in which the signals used propagate. As Einstein said in 1910 (Ref. 11), the "means of sending signals" in his clock adjustment procedure "must be such that we have no reason to believe that the phenomena of signal transmission in the direction *AB* differ in any way from the phenomena of signal transmission in the direction *BA*."

The sphere model implies that this condition is not met in all inertial frames of reference if Einstein's clock adjustment procedure is used. More precisely, it implies that, if the condition is met locally in a first inertial frame of reference Σ, then it is not met in other inertial frames of reference that move relative to the first. The reason is that, in the sphere model, the acceleration of a charged particle from rest in Σ creates anisotropic conditions in the arrangement of sphere surfaces surrounding that particle.

This lack of isotropy does not invalidate any of the results or predictions of special relativity. It merely means that, in special relativity, events with equal time coordinates are in general not simultaneous. This insight has important consequences for the correct interpretation of special relativity. It is for example not the case that, in the framework of special relativity, arbitrary large velocities in Σ would lead to a reversal of cause and effect or time travel into the past, as for example suggested in Ref. 13. Such an impression can only arise if equal time coordinates in special relativity are thought to invariably express a relationship of simultaneity. Similarly, the idea that we live in a "block universe" in which the past and the future are entirely given[17] is a result of the misguided idea that equal time coordinates in special relativity invariably express a relationship of simultaneity.

Finally, the sphere model provides a simple resolution of the Andromeda paradox. In the Andromeda paradox as set out by Penrose,[18] if two people walking past each other both use Einstein-adjusted clocks, they might subsequently disagree by a matter of days over whether an event in the Andromeda galaxy had already occurred when they passed each other. In the sphere model, there is no substance to their disagreement: In general equal time coordinates as defined by Einstein-adjusted clocks do not imply a relationship of simultaneity. Therefore, the two people's time coordinates cannot be used to determine whether or not the event had already occurred by the time of their chance encounter.

The sphere model is fully consistent with special relativity as it leads to exactly the same forces between charges in uniform motion in Σ as classical relativistic electromagnetism. This also means that it is consistent with the principle of relativity as long as Einstein-adjusted clocks are used in all inertial frames of reference. The sphere model can thus be used in any inertial frame of reference in which clocks have been Einstein-adjusted, not just in Σ. However, in frames other than Σ the spheres do not have the same physical significance as in Σ but are just a mathematical device that can be used to produce the empirically correct result.

IX. CONCLUSION

The model presented in this article sheds new light on the nature of the forces between two charged particles q_1 and q_2 moving at constant velocities. The sphere model explains the magnitude and the direction of such forces in terms of changes in the sphere surface arrangements surrounding the charges and associated changes in the flow of information between them. Those changes, which become more pronounced as the speed of the charges in Σ approaches c, are caused by the fact that in Σ electric information is transmitted at the finite speed c.

In the sphere model, the magnitude of the force on q_2 as measured by a co-moving spring balance takes the form of a simple and plausible generalization of Coulomb's law. The direction of the force is a combination of two vectors which represent the directions from which q_1 information arrives at q_2. The forces between moving charges are thus explained in a way which is both simpler and much more plausible than the classical route via the magnetic field, whose origin remains unexplained in classical electromagnetism.

The sphere model is fully consistent with the results and predictions of special relativity. Indeed, it provides a simple and plausible explanation of how the forces between moving charges are transformed in a way which is consistent with length contraction. The sphere model also implies the independence of the speed of electric disturbances from the speed of the source of those disturbances. From this combined with length contraction, the whole apparatus of special relativity follows, provided movement in Σ affects the rate at which any kind of clock ticks in the same way. Finally, the sphere model implies that in general events with equal time coordinates as determined in accordance with Einstein's clock adjustment procedure are not simultaneous. This insight is important for the correct interpretation of special relativity. However, it does not mean that it is necessary or even desirable for physicists to adopt time and space coordinates different from those used in special relativity. This is not just because Einstein clock adjustment leads to the familiar laws of physics in all inertial frames, but also because it is difficult or impossible to know in which frame of reference light signals (or other signals) locally propagate in isotropic conditions. As a result, the sphere model may not change much in the way physicists do physics, but I hope that it can help to deepen our understanding of the physical world.

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